An Olympiad problem appeal

Hans Lausch*

Call for problem donation

Once every three years the Senior Problems Committee of the Australian Mathematical Olympiad Committee (AMOC) turns to our mathematical community at large with an appeal for problem donations that can be used in national, regional and international senior-secondary-school mathematics competitions. The latest appeal [1] provided examples of competition problems that had been set for various contests in Australia and in the Asia-Pacific region between 2001 and 2003. The present article is to repeat this exercise with problems from competitions held between 2004 and early 2007. Problems chosen for these competitions are from 'pre-calculus' areas such as number theory, geometry (with a strong preference for 'Euclidean' geometry), algebra, discrete mathematics, inequalities and functional equations. Here are some examples of problems used in contests held in 2004–2007.

(1) The AMOC Senior Contest is held in August of each year. About 100 students, most of them in Year 11, are given five problems and four hours to solve them. The following problem, Question 1 of the 2004 contest, has received the difficulty rating 'easy-to-medium':

> Consider eight points in a plane consisting of the four vertices of a square and the four midpoints of its edges. Each point is randomly coloured red, green, or blue with equal probability.
>
> Show that there is a more than a 50% chance of obtaining a triangle whose vertices are three of these points coloured red.

(2) Question 2 of the 2004 AMOC Senior Contest turned out to be 'hard':

> Let $a_1, a_2, \ldots, a_{2004}$ be any non-negative real numbers such that $a_1 \geq a_2 \geq \cdots \geq a_{2004}$ and $a_1 + a_2 + \cdots + a_{2004} \leq 1$.
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> Prove that $a_1^2 + 3a_2^2 + 5a_3^2 + 7a_4^2 + \cdots + 4007a_{2004}^2 \leq 1$.

(3) The Australian Mathematical Olympiad (AMO) is a two-day event in February with about 100 participants. On either day, students are given a four-hour paper containing four problems. Students found the following problem, Question 5 of the 2006 AMO, quite easy:

> Let $ABCD$ be a square, and let E be a point on its diagonal BD. Suppose that O_1 is the centre of the circle passing through ABE and O_2 is the centre of the circle passing through ADE.
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> Show that AO_1EO_2 is a square.

*School of Mathematical Sciences, Monash University, VIC 3800
E-mail: hans.lausch@sci.monash.edu.au

(4) Here is a 'medium-to-hard' geometry problem from the 2007 AMO (Question 7):

> Let ABC be an acute angled triangle. For each point M on the segment AC, let S_1 be the circle through A, B and M, and let S_2 be the circle through M, B and C. Let D_1 be the disk bounded by S_1, and let D_2 be the disk bounded by S_2.
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> Show that the area of the intersection of D_1 and D_2 is smallest when BM is perpendicular to AC.

(5) Note that the difficulty ratings of the above problems were arrived at from hindsight. In reality, problems committees often find it rather hard to pin down the difficulty of a problem when setting it. This phenomenon is well known not only on national competitions but also in regional and international competitions. Here is an example.

The Asian Pacific Mathematics Olympiad (APMO) takes place in March. The contest is a four-hour event with five problems to be solved. About 20 countries, most of them from the Pacific Rim, take part in the APMO. Usually, 25 to 30 Australian students are invited to participate in this competition. At the 2006 APMO, Question 5 was considered by the APMO Problems Committee, which includes mathematicians from three different countries, as the most difficult one on the paper. A significantly large percentage of participants, however, contradicted the committee by providing perfect or nearly perfect solutions. Here is the problem, which had been submitted by the Australian problems committee:

> In a circus there are n clowns who dress and paint themselves up using a selection of 12 distinct colours. Each clown is required to use at least five different colours. One day, the ringmaster of the circus orders that no two clowns have the same set of colours and no more than 20 clowns may use any one particular colour.
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> Find the largest number n of clowns that the circus could employ so as to make the ringmaster's order possible.

Problems of recent International Mathematical Olympiads continue to be reported regularly in the *Australian Mathematical Society Gazette*. The complete set of AMO problems and solutions covering the period 1979–1995 can be found in [2], whereas the problems and solutions of all APMOs between 1989 and 2000 have appeared in [3], while the problems, including solutions and statistics, of each year's AMOC Senior Contest, the AMO, the APMO, the International Mathematical Olympiad and some intermediate-secondary school mathematics competitions are available in the AMOC's year books [4].

Problem donations will be gratefully received by me as Chair of the AMOC Senior Problems Committee and credit to the donor of successful problems will be given in [4].

References

[1] Lausch, H. (2004). In quest of Olympiad problems. *Aust. Math. Soc. Gazette* **31,** 170–171.

[2] Lausch, H. and Taylor, P. (1997). *Australian Mathematical Olympiads 1979–1995.* Canberra, Australian Mathematics Trust.

[3] Lausch, H. and Bosch Giral, C. (2000). *Asian Pacific Mathematics Olympiads 1989–2000.* Canberra, Australian Mathematics Trust.

[4] Storozhev, A.M., Henry, J. B. and Di Pasquale, A. *Mathematics Contests: The Australian Scene.* (Appears once a year.)