Book reviews

Mathematics for Multimedia

Mladen V. Wickerhauser
Academic Press 2004, ISBN: 0-12-748451-5

A good book should have good content, which has been carefully selected for the target audience and presented in an interesting and easy-to-read way. The content of this book is excellent. It covers main mathematical topics common and essential to multimedia. As a computer scientist working with a group of research students in the area of multimedia databases, it has been my view for some time that today's computer science graduates are not mathematically prepared to work in the area of signal processing, image processing and multimedia in general. So I studied this book with great interest. It did not take very long to disappoint me.

This is not a book I would recommend to a computer science PhD student who wants to work in multimedia research but is frustrated by many mathematical challenges involved in this area. This book is really about mathematics only, not about multimedia, not even attempting to show the links between the mathematical topics introduced and how they can be applied in multimedia.

One example is when discussing lossless encoding. One would expect an introduction to the famous Shannon's Information Theory, and some applications of entropy encoding in image compression. None of these are mentioned, not even in the suggested reading list. Another example is the part introducing DCT, which is used in JPEG encoding. This application is interesting to computer science students, and can motivate them to study this book and to understand the content better. Again, this book has discussed nothing more than DCT itself.

The publisher advertises this book as a book for 'computer science and multimedia students and professors'. It is not. It does not attempt to relate to the mathematical 'pearls' discussed in the book to the background and questions the targeted readers may have. Although it manages to spare a few pages to discuss binary representation of numbers and the definitions of graphs and trees, which are basic concepts all computer science students know very well.

Some algorithms in C are given in the book. As a computer science person, I would prefer to see some examples of the concepts and properties which sometimes I find difficult to understand, instead of some C code (not to mention different versions of the code using recursions or only iterations). After all, there are plenty of efficient libraries available. It is
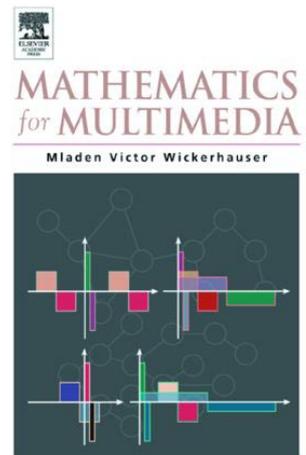
also interesting to comment on the first algorithm given in the book (Euclid's Algorithm). The code is correct, but quite difficult to understand, as one pre-condition is $a \geq b$, but the first iteration of the code is just to reverse this condition (by using $b\%a$). This algorithm can be represented in a much simpler way that is also easier to understand.

I recommend the book *Fundamentals of Multimedia* by Z.-N. Li and M.S. Drew (Pearson Education, 2004). *Fundamentals of Multimedia*, although not as concise and precise as *Mathematics for Multimedia*, is very easy for computer science students to read, and covers many topics discussed in this book.

Xiaofang Zhou
School of Information Technology and Electrical Engineering, University of Queensland, QLD 4072.
E-mail: zxf@itee.uq.edu.au

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Divine Proportions:
Rational Trigonometry to Universal Geometry

N.J. Wildberger
Wild Egg, Kingsford, 2005, ISBN: 0-9757492-0-X 00-01 (51-01 97-01)

The book introduces an alternative approach to treating trigonometry and the Euclidean geometry of the plane. In the setting of the usual model of Euclidean geometry, the tools involved are not new, and the point is really a new emphasis. In particular at the core are *quadrance*, measuring the separation of points, and *spread*, measuring the separation of lines. In the usual setting and framework of Euclidean geometry, quadrance is exactly the square of the distance between points while spread agrees with the square of the sine of the angle between lines. The idea is that these measures should be taken as the basic building blocks of the theory, from both the conceptual and practical (i.e. calculational) point of view.

An observation made very early in the work is that the 'quadratic quantities' (to borrow a term from the author) of spread and quadrance may be used to develop a theory of trigonometric type calculations that avoid taking roots or using trigonometric functions. This is the 'rational trigonometry' from the title, and is so named because if calculations are handled suitably then they involve only rational expressions in the input number data. The latter point is central to the entire development since it means that many theorems may be given in a form (and are in this text) that is valid over an arbitrary number field (of characteristic other than two). The same idea is used to develop a version of aspects of Euclidean geometry that is valid over arbitrary number fields, and this is termed 'universal geometry'.

The first part of the book gives an overview of the rational trigonometry approach and its applications, and then in the next section this topic is developed more fully. Here, many of the classical elementary theorems are rephrased and recovered (but over a general field) using the new building blocks. For example, an old favourite from school days, the perpendicular bisector of a line, is seen, via a theorem, to be an 'equal quadrance to two points'; that is, all

points on the bisector are of equal quadrance to the end points on the bisected line segment. Geometry is covered in the third part of the text, and here we find an algebraic and field independent treatment of a theory which generalises a significant part of the classical theory of triangles, polygons, circles, conics and tangents to arbitrary fields. The final part of the book deals with basic problems in a variety of directions ranging from projectile motion and surveying through to a discussion of rational replacements for spherical coordinates. While this section is called 'Applications', it really collects worked examples indicating how to treat a range of questions using the machinery of quadrance and spread.

As one might expect in a book which is claiming to challenge the traditional thinking, a large part of the work is concerned with justifying the theory. This is particularly true in the early sections where there is discussion pointing to claimed failings and inadequacies of past approaches and comparisons between the ideas in the text with what is asserted to be the traditional treatment. A worked example compares the approaches by calculating a certain section across a triangle in two ways. One treatment uses trigonometric functions and their inverses to get (via a calculator) an approximate solution while the other uses the rational techniques to obtain an exact solution in surd form. However as the author himself points out other classical approaches may be used that avoid calculating angles and will equally yield the exact solution. Indeed the rational trigonometry approach is just a collection of steps in Euclidean geometry! In the move to the language of 'Universal Geometry', some standard theorem statements become simpler while others become more complicated. This mostly occurs in an obvious way depending on whether the classical theorem is linear, quadratic or otherwise in nature. Among the most obvious, Pythagoras' theorem is a linear relation of quadrances. On the other hand, if the quarance between points A and B is written $Q(A,B)$, then in terms of rational trigonometry the collinearity of points A, B, and C is characterised by the so-called 'triple quad formula' $(Q(A,B) + Q(B,C) + Q(A,C))^2 = 2(Q(A,B)^2 + Q(B,C)^2 + Q(A,C)^2)$. This is hardly as intuitively appealing as the usual Euclidean notion of 'between': the point B is between A and C if and only if the distances between A and B, and between B and C add to the distance from A to C. On the other hand the symmetry in the triple quad formula means that we obtain an analogous result but where an ordering of the points is not needed.
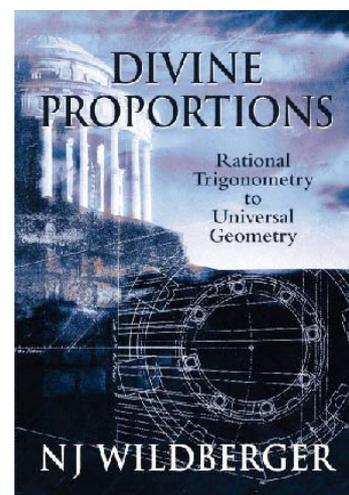
Strictly speaking very little background is assumed of the reader and the material in the book is almost entirely elementary. Although most of the material could be treated over the reals the book would be confusing to a student not yet comfortable with the complex numbers. For example a 2-*proportion* is defined to be an expression $a : b$, for numbers a, b, with $a : b$ taken to be equal to $\lambda a : \lambda b$ for any non-zero λ in the field. Extending this notion in an obvious way, a line is defined to be 3-proportion $l = \langle a : b : c \rangle$ (with a, b not both zero), and this is said to be *null* if $a^2 + b^2 = 0$. In reality it seems unlikely that the book will appeal to the mathematically immature. For example, general fields (of characteristic other than two) are used throughout and there is significant emphasis on the applications to geometry over finite number fields, but fields are not defined formally. Also the simplifications that rational trigonometry brings to some calculations will mainly be

evident to those who can appreciate the redundancies involved in the usual calculations via trigonometric functions and their inverses. To add to this viewpoint it should be pointed out that the style is extremely informal. Although there is a claim to develop Universal Geometry there is no definition of geometry in this text. Rather it is taken as a notion understood by the reader and what is presented in the section on Universal Geometry is simply a collection of theorems which are geometric in nature.

The cover of the book asserts 'This unique and revolutionary text establishes new foundations for trigonometric and Euclidean geometry'. There are more statements along those lines in the preface and introductory sections. Is there really a revolution within? The standard *model* of the Euclidean plane is the affine plane equipped with the standard quadratic form (the usual dot product). In this model, angle and length are not the basic building blocks but are derived quantities. If we make this our starting point then the move to the rational trigonometry approach is a gentle shift in emphasis rather than a revolution. This shift is concerned with how to deal with proofs, rather than with shaking foundations. I do not believe that teachers, students, engineers and other geometry consumers will be willing to discard the intuitive and, I would say, conceptually powerful notions of length and angle. For example, angle is not an arbitrary measure of the separation of lines but one which reflects the isotropy of the Euclidean plane. Experiments indicate that angular momentum is a conserved quantity. An upshot is that motions with constant angular velocity are common and important in our environment. The rotation of the earth is a salient example. It seems such motions will not yield a simple description in terms of spread.

These comments are not meant to imply that the book is without valuable ideas. A small shift can have major implications. Although the means to avoid calculating angles explicitly (for many calculations) is already available in Euclidean geometry, the text does provide a brief list of 'laws' and formulae which enable this to be done systematically. It makes sense to welcome these notions, these carefully packed parcels of information, into the Euclidean language. They may be manipulated in the same way as the classical laws. As indicated in the later sections this extension of the language enables many of the classical theorems of circles, triangles and so forth to be formulated in a universal and concise way that makes sense irrespective of the underlying field.

Rod Gover
Department of Mathematics, The University of Auckland, Private Bag 92019, Auckland, New Zealand
E-mail: r.gover@auckland.ac.nz

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A Generating Function Approach to the Enumeration of Matrices in Classical Groups Over Finite Fields

J. Fulman, P.M. Neumann and C.E. Praeger

This volume studies the proportions of cyclic, separable, semisimple, and regular matrices in a classical group defined over a finite field.

Let A be a matrix defined over a field F.

- A is *cyclic* if its characteristic polynomial $c(x)$ equals its minimal polynomial $m(x)$.
- A is *separable* if $c(x)$ has no repeated roots in the algebraic closure of F.
- A is *semisimple* if $m(x)$ is separable.
- If F is finite and A is an element of a classical group G defined over F, then A is *regular* if its centralizer in the corresponding algebraic group over the algebraic closure of F has dimension equal to the Lie rank of G.

The classical groups considered in the volume are $\mathrm{GL}(n,q)$, the group of all invertible $n \times n$ matrices with entries in $\mathrm{GF}(q)$; the symplectic group $\mathrm{Sp}(n,q)$, the subgroup of $\mathrm{GL}(n,q)$ consisting of all matrices which preserve a nondegenerate bilinear form; the orthogonal group $\mathrm{O}^{\epsilon}(n,q)$, the subgroup of $\mathrm{GL}(n,q)$ consisting of all matrices which preserve a nondegenerate quadratic form; and the unitary group $\mathrm{U}(n,q)$, the subgroup of $\mathrm{GL}(n,q^2)$ consisting of all matrices which preserve a nondegenerate sesquilinear form. These families of matrix groups and their related simple quotients are fundamental objects of study.

Let A be an element of a classical group defined over a finite field F. A cyclic matrix A has the property that the vector space of $1 \times n$ matrices over F is cyclic as an $F\langle A\rangle$-module. In almost all cases, A is cyclic if and only if it is regular; A is semisimple if and only if it has order coprime to the characteristic of F; and A is separable if and only if it is regular and semisimple.

This volume complements and extends earlier work. Fulman [2] and Wall [9] independently used generating functions to study the proportion of cyclic and separable matrices in $\mathrm{GL}(n,q)$. They proved that, as $n \to \infty$, the limiting proportions of cyclic and separable matrices in $\mathrm{GL}(n,q)$ are $(1-q^{-5})/(1+q^{-3})$ and $1-q^{-1}$ respectively.

Neumann & Praeger [6] obtained estimates for the proportion of cyclic and separable matrices in $\mathrm{M}(n,q)$, the set of all matrices with entries in $\mathrm{GF}(q)$. In [7] they extended these results for cyclic matrices to the general classical groups, which preserve the appropriate form up to scalar multiple. Guralnick & Lübeck [5] have also determined the proportions of separable matrices in classical groups and the exceptional families of groups of Lie type. Their combinatorial and geometric approach is very different and their results are useful only for fields of size at least 5.

The authors extend the results of Fulman and Wall on cyclic and separable matrices in $\mathrm{GL}(n,q)$ to the other classical groups and also obtain similar results for the classes of regular and semisimple elements. For each classical group G of dimension n defined over $\mathrm{GF}(q)$, and for each of the four named types of matrices, they obtain very precise estimates for the probability $p(n,q)$ that a random element of G is of this kind. They focus on the case where q is fixed and n is allowed to grow. In almost all cases, $p(n,q)$ has the form $1-aq^{-1}+b(n)q^{-2}$ or $1-aq^{-3}+b(n)q^{-4}$, where a is a constant depending on the type of the matrix and the classical group, and $b(n)$ depends on n but is bounded above and below independently of n for sufficiently large n. Observe that, for fixed n, these probabilities tend to 1 as $q \to \infty$. They prove the existence of the limit $p(\infty,q)$ as $n \to \infty$, obtain precise expressions for this limit as a function of q, find sharp lower and upper bounds, and estimate its rate of convergence. The results are useful for all finite fields.

The central components studied are power series generating functions expressed as infinite products, and characteristic and minimal polynomials for elements of classical groups. The results of Fulman and Wall demonstrate that the limiting probabilities for cyclic and sepa-

rable matrices in $GL(n, q)$ are rational functions of q. It is not yet known which, if any, of the remaining limits have this property.

Recently, Britnell (see, for example, [1]) has extended these techniques to some of the other classical groups not covered here, obtaining similar results.

An important motivation for this work is the use of such elements in the design and analysis of algorithms for the study of subgroups of $GL(n, q)$. For example, Neumann & Praeger [8] have exploited cyclic matrices to obtain a variation of the MEATAXE, a fundamental algorithm to decide if an FG-module fixes any proper subspace of the underlying vector space. Other applications include the study of fixed point free permutations (derangements) in finite transitive permutation groups [3] and monodromy groups of curves [4].

A major strength of this volume is its precision. The techniques are powerful, providing detailed insight into the structure of the classical groups. The volume is very well written and its material is both clearly presented and summarized. It contains a wealth of important results and represents a major contribution to our understanding of these groups.

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E.A. O'Brien

Department of Mathematics, The University of Auckland, Private Bag 92019, Auckland, New Zealand
E-mail: obrien@math.auckland.ac.nz