Book reviews

Elements of real analysis

M.A. Al-Gwaiz and S.A. Elsanousi
Chapman and Hall/CRC, 2007, ISBN 1-58488-661-7

This new text provides a readable and solid treatment of real analysis. It is the basis of a two semester sequence taught at King Saud University. It would be ideal as a class text or for independent learning.

The style is friendly, maybe a little too wordy sometimes in motivating concepts, but students might appreciate this. The translation from Arabic is fine. On page 69 in an example on continuity we are told that *given* $\epsilon > 0$ *our burden is to find* $N \ldots$ which I thought was a nice way to put it. Limit points are called *cluster points*.

There is a decent spattering of exercises throughout which are at a good level, not too hard to start with. There are no solutions provided, which can sometimes be a drag for students wanting some positive reinforcement. The CRC Press website says there is an instructors solution manual. There are lots of worked examples in the text which demonstrate the techniques required.

I found the first two chapters to be a bit of a chore: some set theory which could be skipped, the reals defined axiomatically, loads of tedious results that come straight from the definitions. I would have preferred a more punchy start — for example, Rudin [2] starts by constructing reals from rationals using Dedekind cuts — something a bit different for students who have just progressed from years of calculus. Also Rudin defines metric spaces early on and frames his text in this generality.

But it definitely picks up from there. The rest of the chapters follow the standard treatment: sequences and series of reals, limits and continuity, differentiation, the Riemann integral, then sequences and series of functions. The final two chapters cover Lebesgue measure and integration carefully and rigorously and will appeal to more advanced students. They justify why the Riemann integral isn't sufficient.

Another recent text is Morgan [1], a short and direct treatment of all the highlights and cool examples, very readable and a good supplementary text students might really like. And relatively inexpensive.

References

[1] Morgan, F. (2005). *Real Analysis and Applications: Including Fourier Series and the Calculus of Variations.* American Mathematical Society.
[2] Rudin, W. (1964). *Principles of Mathematical Analysis.* McGraw-Hill Book Company, Inc., New York.

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Numbers at work: A cultural perspective

Rudolf Taschner (Translated by Otmar Binder and David Sinclair-Jones)
A.K. Peters Ltd, 2007, ISBN 1-56881-290-6

There can be no better way to begin this review than by quoting a sentence from the Translator's Note:

> Reading *Numbers at Work* is like walking down a great corridor lined with books: there is much to see on the shelves, much to learn from each individual book, and great benefit to be gained from reaching the end.

Incidentally, the translation team, Otmar Binder and David Sinclair-Jones, are to be congratulated on turning the original German text into clear and idiomatic English while preserving the intellectual spirit of the original.

Rudolf Taschner, named Scientist of the Year by the Austrian Association of Science Journalists in 2004 for his pioneering efforts to propagate mathematics to a broad public audience, is a mathematics professor at the Technical University of Vienna. This book began in 2003 as a series of presentations and workshops at the Vienna MuseumsQuartier intended to foster public awareness of mathematics as a cultural achievement. Published in German by Vieweg as *Der Zahlen gigantische Schatten* (The Gigantic Shadows of Numbers), it received wide critical acclaim and we owe thanks for the English version to the efforts of publishers A.K. Peters.

As a mathematician with interests in history, I did not expect to learn a great deal from such a popularisation, but was pleasantly surprised to have my expectations dashed. There are many gems to be found in this book that I have not seen elsewhere, and we owe a great deal to the formidable erudition of Professor Taschner.

For example the first chapter, on the early numerical works of the Greeks, compares the numerological mysticism of the Old Testament and the Kabbalah to that of the Pythagoreans. The second, ostensibly concerned with the patterns of intervals to be found in the works of Bach, presents a clear discussion of the difficulties of tuning a keyboard so that an octave is divided into 12 almost equal semitones. Mathematically, this boils down to inserting a geometric series of 12 terms between 1 and 2, and then finding the best rational approximation to each term.

Another chapter compares the concept of time among philosophers, physicists, (notably whether it makes sense to talk about what happened before the Big Bang), and poets, with particular reference to the work of Hugo von Hofmannsthal, the librettist of *Der Rosenkavalier.*

And so it goes: in the chapter devoted to metrisation of space we learn not only about Cartesian co-ordinates but also about geographical projections. In that concerned with numerical representation, the author considers not only the well known (even to the intended audience) decimal and binary systems, but also the ternary system, which has unexpected advantages over both of these. We even learn how it may be implemented in electronic systems.

In spite of my overall enthusiasm, I have some negative feelings about this volume. To mention a few, the explanation of Gödels *Incompleteness Theorem* is very curt and may be incomprehensible to anyone who is not well versed in the topic. Furthermore, Professor Taschner ignores advances in analysis and set theory of the past half century and objects to anyone who dares to mention infinity and numbers in the same breath. For example, he claims Dedekind was guilty not just of faulty logic, but of immorality for daring to calculate with infinite quantities! A related misgiving I have is that the author identifies any real number with its decimal (or binary or ternary) representation. This leads him to questionable claims about the mystery of π and other irrational numbers; for example, that they are forever beyond the reach of human knowledge.

A few typographical errors caught my eye. They scarcely detract from an otherwise finely produced volume. The many well-chosen and properly credited coloured illustrations add to the readability of the text. So too do the many pages of Notes and the adequate Bibliography and Index.

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Experimental mathematics in action

David H. Bailey, Jonathan M. Borwein, Neil J. Calkin, Roland Girgensohn, D. Russell Luke and Victor Moll
A.K. Peters Ltd, 2007, ISBN 978-1-56881-271-7

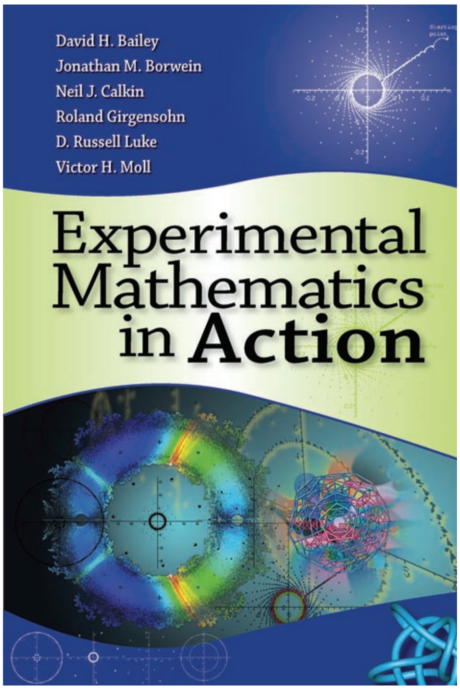
This book originated from a short course of the same name. It is perhaps true that, as the authors state in the preface:

> The last twenty years have been witness to a fundamental shift in the way mathematics is practiced. With the continued advance of computing power and accessibility, the view that “real mathematicians don’t compute” no longer has any traction for a new generation of mathematicians that can really take advantage of computer aided research, especially given the modern computational packages such as Maple, Mathematica, and Matlab.

More and more evidence shows that the power of modern computers matched with that of modern mathematical software and the sophistication of current mathematics is changing the way we do mathematics.

Through eight examples, the authors re-emphasise the following roles for computation in Chapter 1: gaining insight and intuition, or just knowledge; discovering new facts, patterns, and relationships; graphing to expose mathematical facts, structures or principles; rigourously testing and especially falsifying conjectures; exploring a possible result to see if it merits formal proof; suggesting approaches for formal proof; computing replacing lengthy hand derivations; and, finally, confirming analytically derived results. Certainly, computer packages can make concepts more accessible.

An overview of some algorithms for experimental mathematics is presented in Chapters 2 and 3. In Chapter 2, the following topics are addressed through a number of examples. First, the integer relation detection methods, for instance PSLQ. These are very often employed in applications to recognise a mathematical constant whose numerical value can be computed to at least moderately high precision, with examples ranging from the BBP formula to applications in quantum field theory. The computation of multivariate zeta values and finally the Ramanujan-type elliptic series are described. In addition, Chapter 3 presents several interesting examples, covering prime number computations, roots of polynomials, various types of numerical quadrature approaches and infinite series summations.



Chapter 4 turns from questions or problems in pure mathematics to those in applied mathematics, particularly to the data analysis side of experimental mathematics. The authors concentrate mainly on the inverse problem of determining the model from an observation. The inverse scattering technique highlights the role of computational experimentation in the development of mathematical methodologies.

More interestingly, a number of 'strange' functions which could be explored on the computer are introduced in Chapter 5. The authors discuss some general properties of a general form of the Weierstrass no-where differentiable functions. Beginning with a system of functional equations, they provide a necessary and sufficient condition for the Weierstrass functional system to have a unique continuous solution. Here the computer can help us visualise the recursive solutions of the system, to point us in the right direction. An infinite Bernoulli convolution will lead to a Bernoulli measure. With the help of the computer to display different curves of different functions and different measures, the authors also show that the measure is singular under one kind of condition while it is absolutely continuous under another condition. In both cases, computer graphs help figure out the direction of mathematical structures and principles. This is extremely important to shorten the time spent on research.

The first paragraph of Chapter 6 summarises:

> Mathematics is often presented as a fait accompli, a finished result, with little attention being paid to the process of discovery or to the mistakes made, wrong turns taken, etc. However, especially where experimental mathematics is concerned, we can learn a great deal from experience,

> both our own and that of others. New directions can be suggested by previous mistakes.

A case-study on factoring integers using a random vector approach is discussed in this chapter. The authors turn one of the factorisation algorithms, the Quadratic Sieve, into an algorithm that produces a sequence of vectors with entries distributed in a quasi-random fashion. This chapter demonstrates that the behaviour of the Quadratic Sieve can be predicted based on the behaviour of a random model if the probabilistic model is chosen appropriately by experimenting.

The evaluation of definite integrals is one of the most intriguing topics of elementary mathematics. Nowadays, with the increasing application of mathematics in financial engineering, the evaluation, especially the numerical evaluation of definite integrals has a promising and important place in modern finance. Chapter 7 introduces the methods of experimental mathematics in the context of definite integrals. The authors emphasise the use of *Mathematica* in their approach to find various definite integrals and their properties. Hence, a number of definite integral evaluations are demonstrated using *Mathematica*.

Several real-life and pure-maths applications are provided in Chapter 8. Examples range from visual computing, visualising DNA strands, chaos games to Hilbert's inequality and Witten's Zeta Function. These successful applications of experimental mathematics very well conclude this fantastic book, with strong evidence of the increasing usefulness of computers in experimental mathematics.

The first eight sections of Chapter 9 contain exercises generally relating to the previous corresponding chapters, while the ninth presents a large compendium of extras. As the authors state in this chapter, these exercises

> ... do provide an interesting opportunity to see how the art of problem solving changes in the presence of modern computers and good mathematical software. In each case, a few lines of computer algebra code either provides the solution, suggests an approach, or at least confirms the answer.

David H. Bailey *et al.* have done a fantastic job to provide very comprehensive and fruitful examples and demonstrations on how experimental mathematics acts in a very broad area of both pure and applied mathematical research, in both academic and industry. Anyone who is interested in experimental mathematics should, without any doubt, read this book!

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